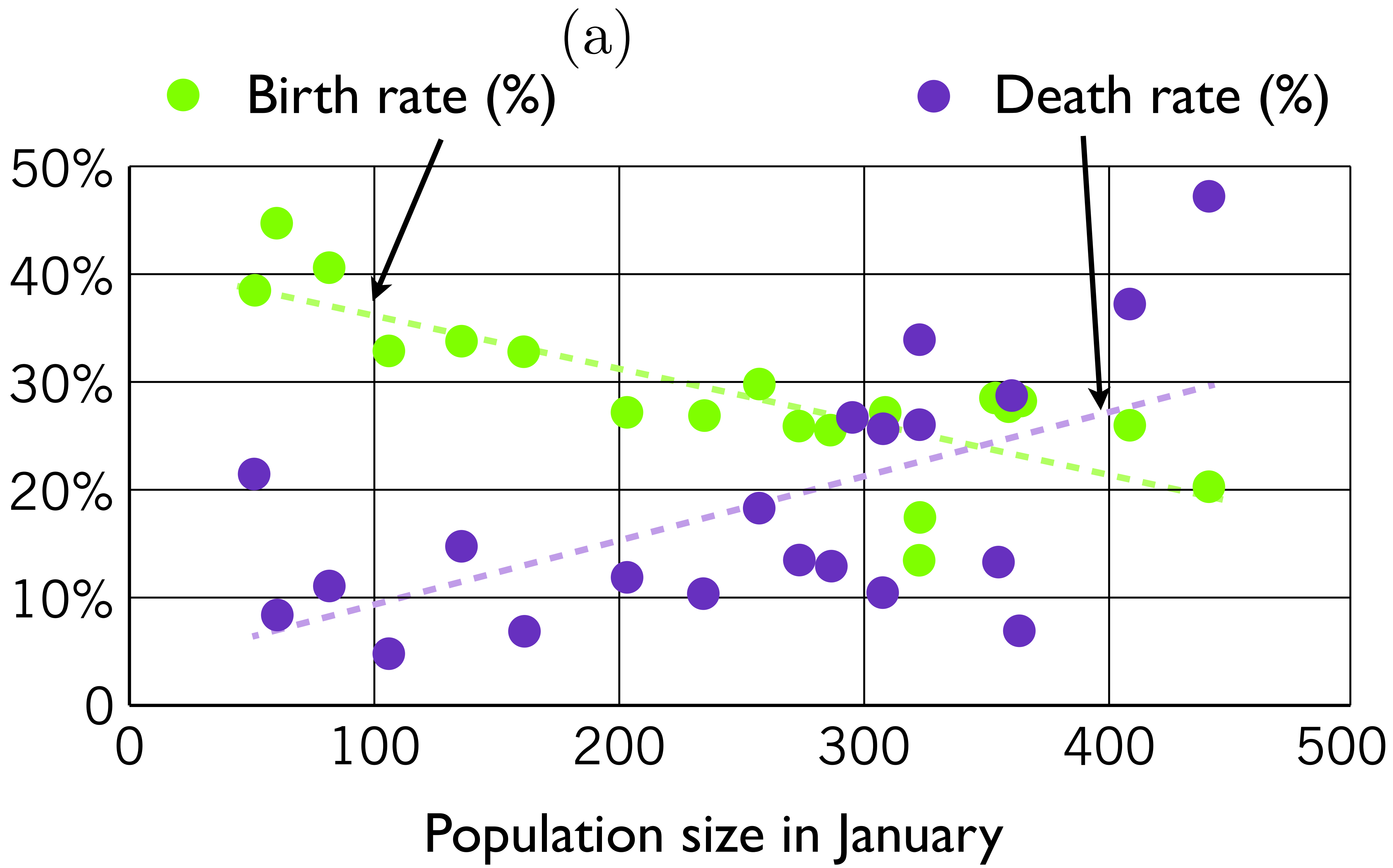
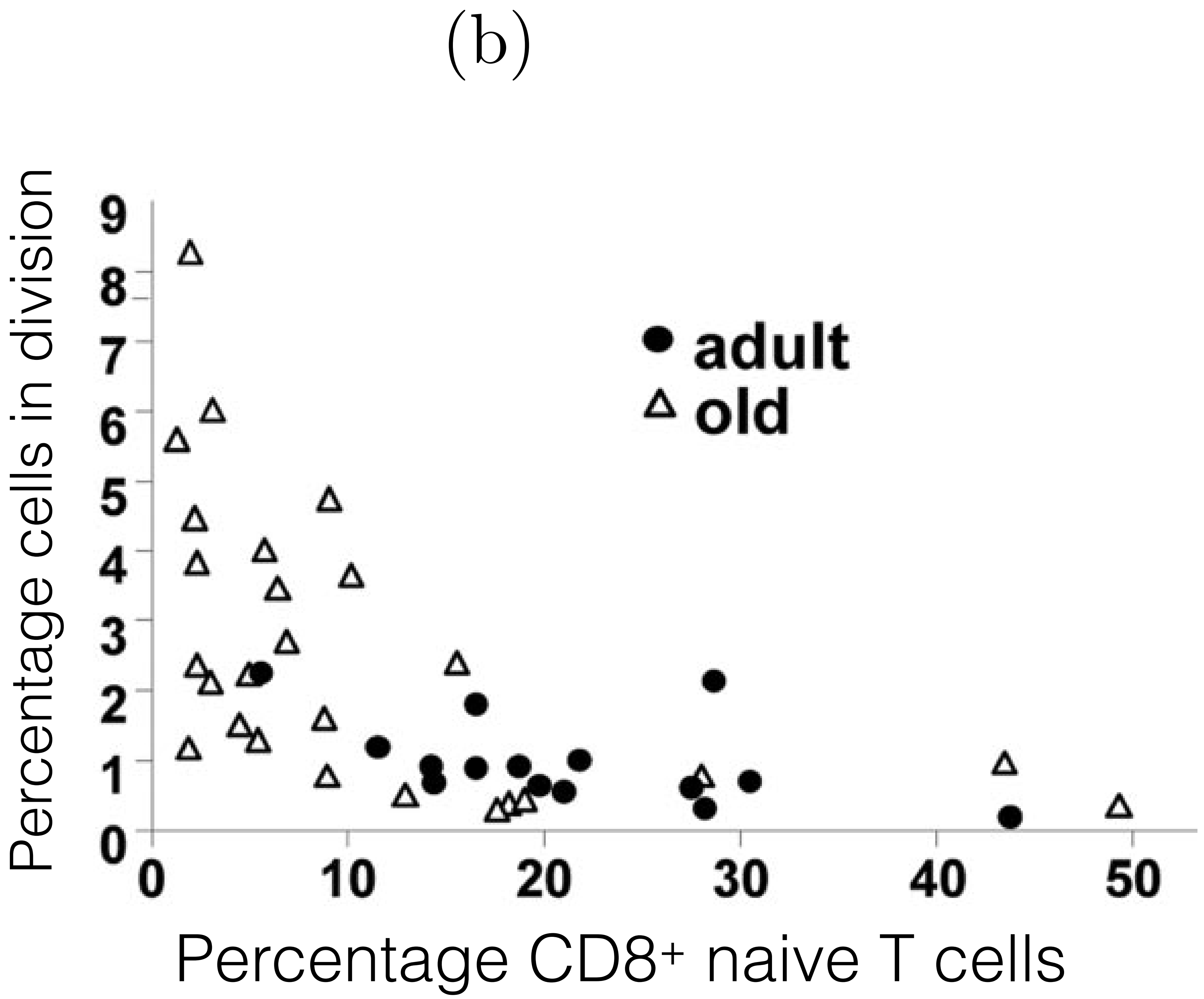
Chapter 3: Density dependence





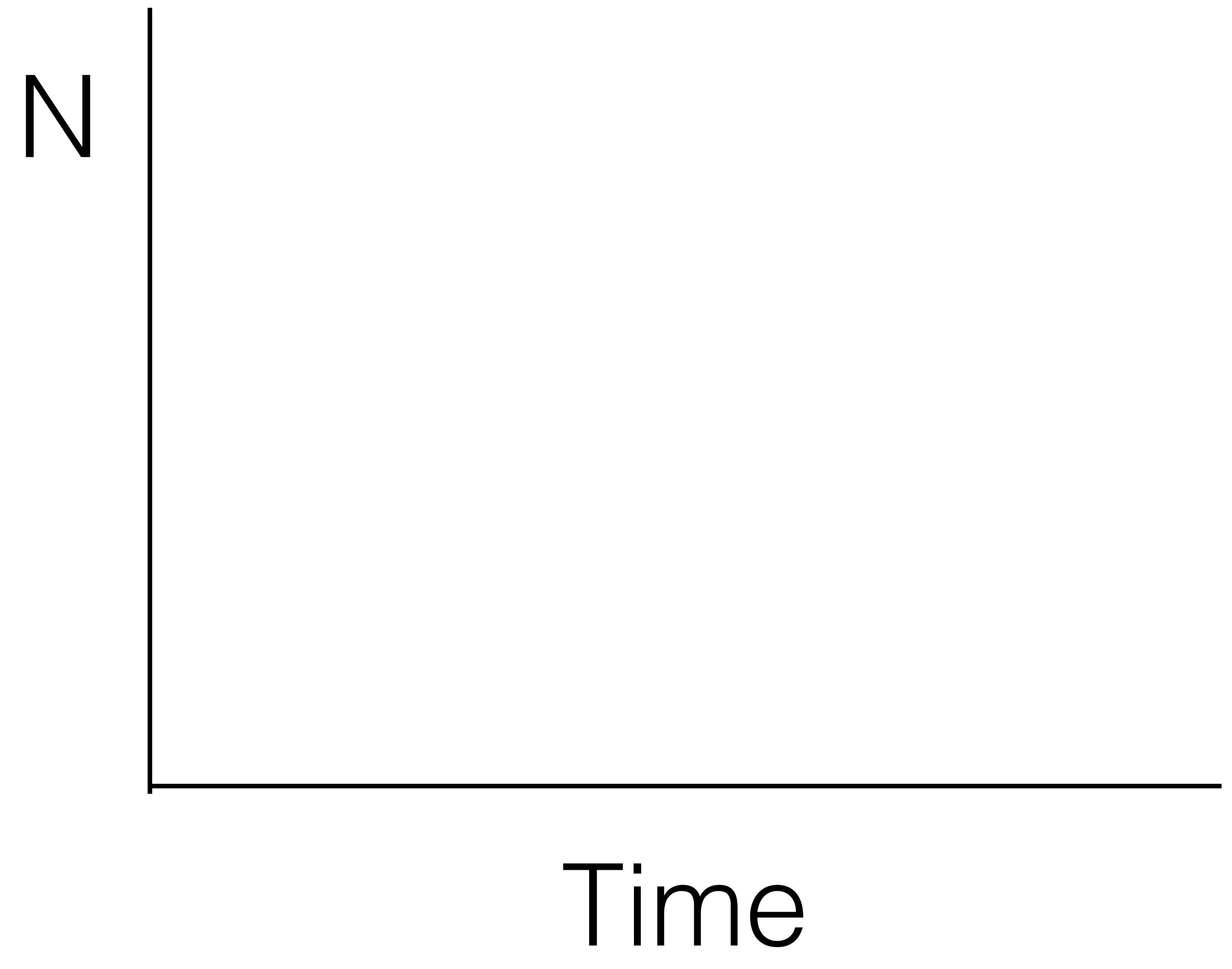
$$\frac{\mathrm{d}N}{\mathrm{d}t} = sf(N) + [bg(N) - dh(N)]N$$

Populations change by immigration, birth, and death processes, which could all depend on the density of the population itself

Typically source death or Logistic growth

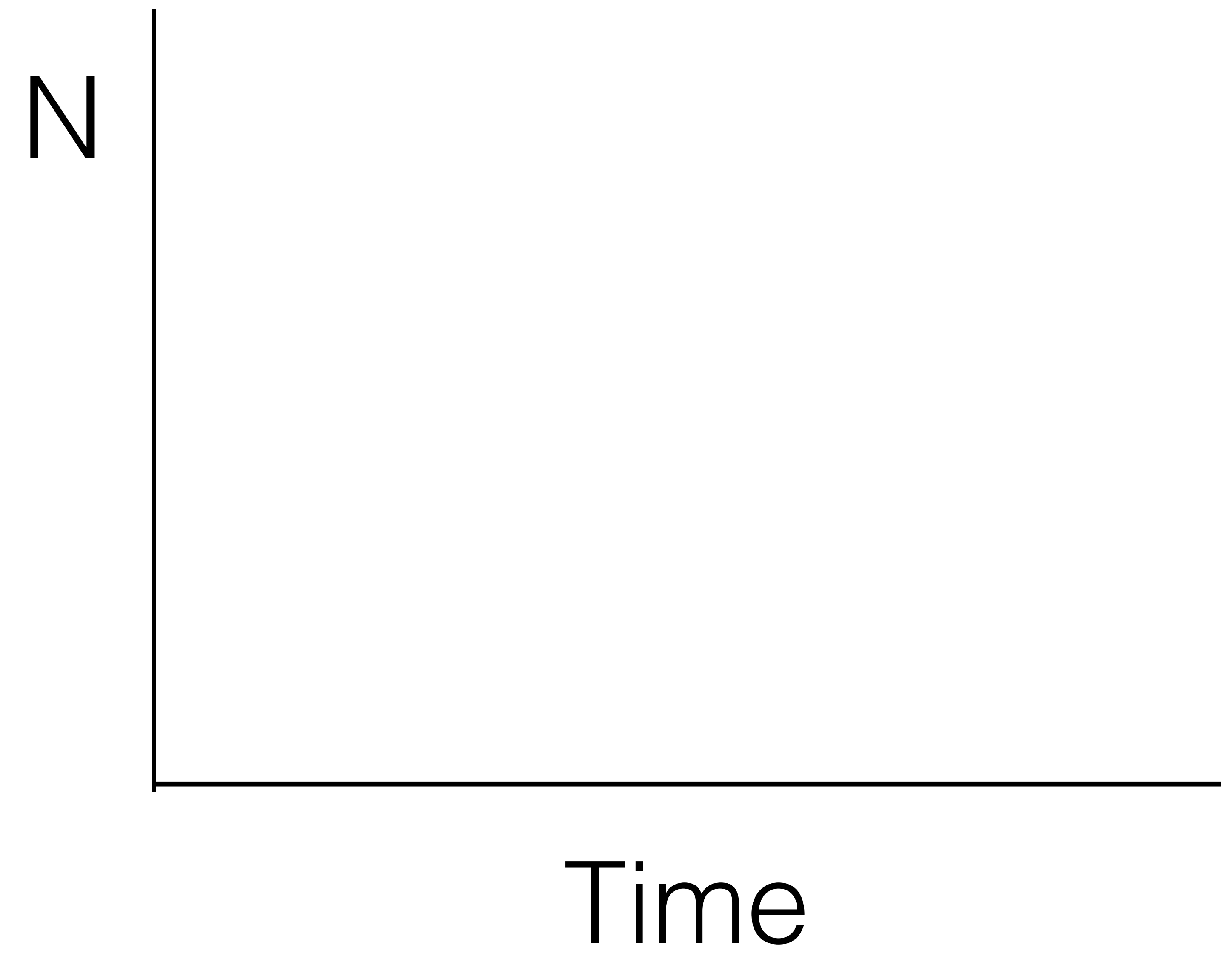
Logistic growth

$$\frac{\mathrm{d}N}{\mathrm{d}t} = rN(1 - N/K) \quad \text{with} \quad \bar{N} = 0 \quad \text{or} \quad \bar{N} = K$$

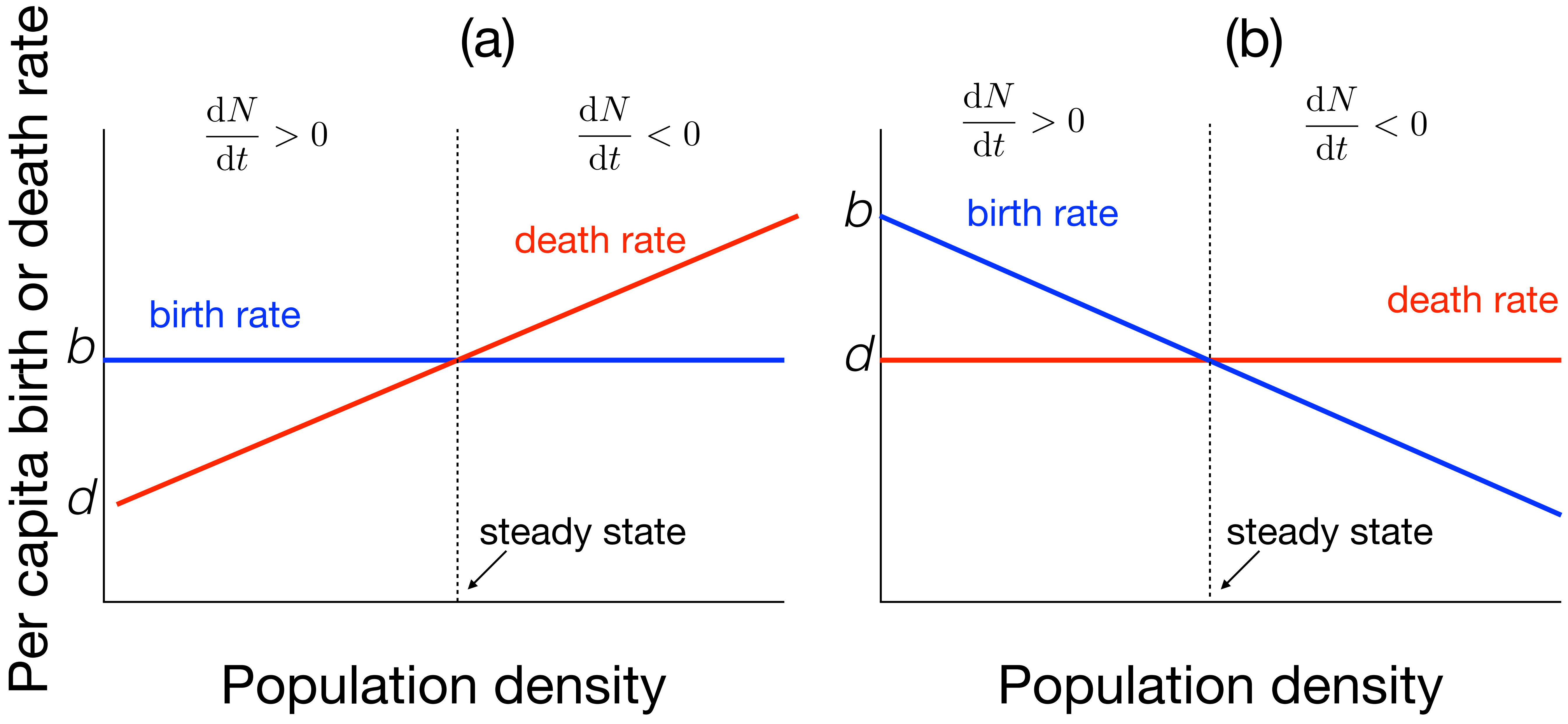


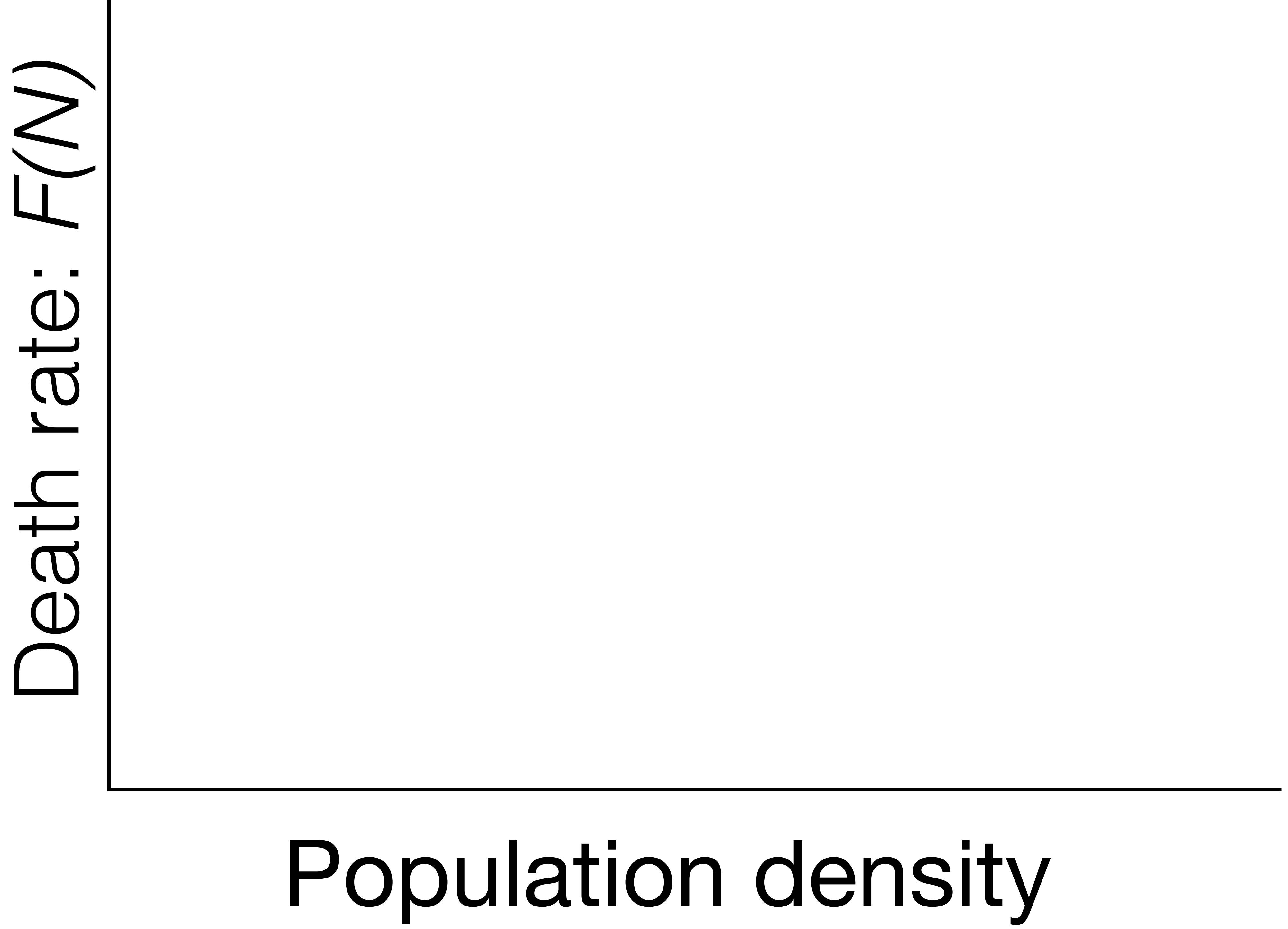
Source death

$$\frac{\mathrm{d}N}{\mathrm{d}t} = s - dN \quad \text{with} \quad \bar{N} = \frac{s}{d}$$



$$\frac{\mathrm{d}N}{\mathrm{d}t} = sf(N) + [bg(N) - dh(N)]N$$



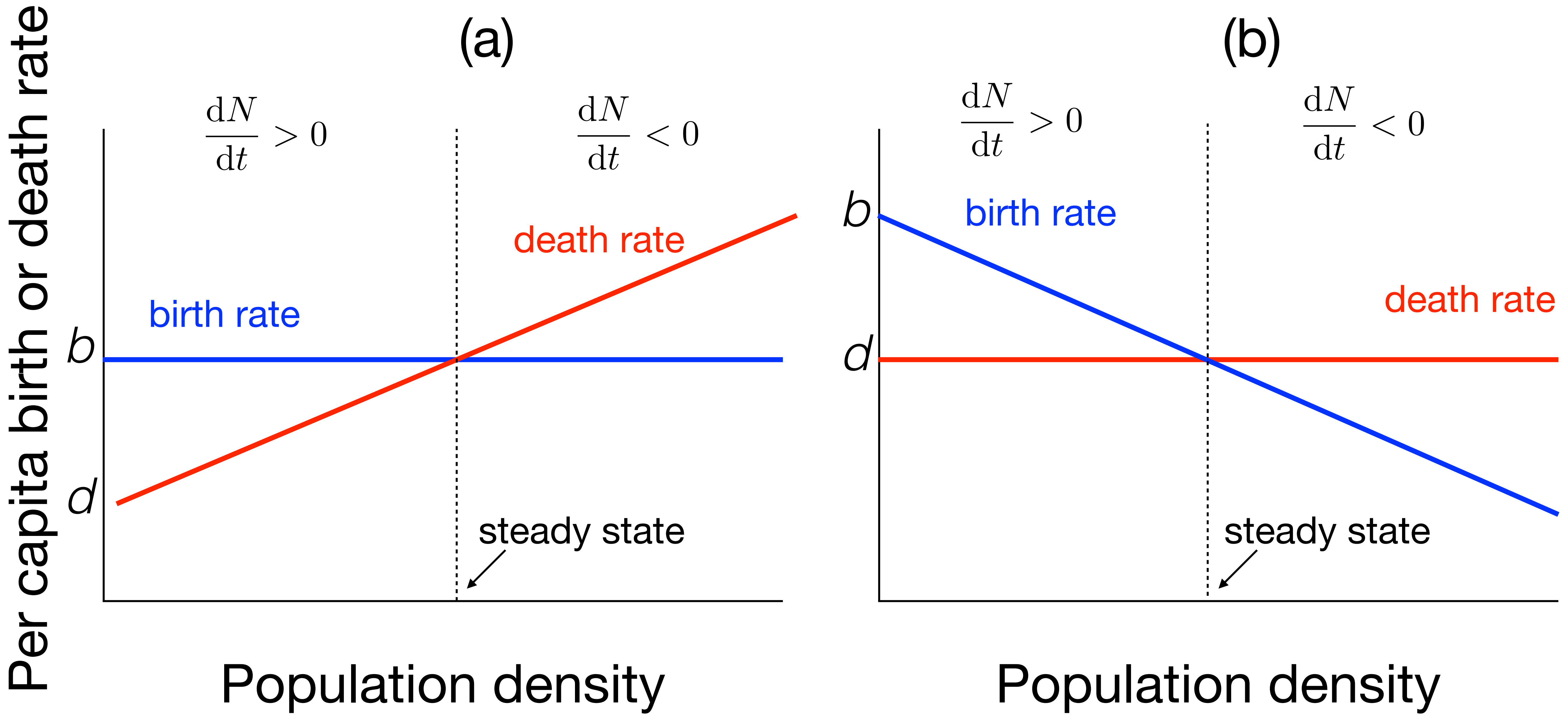


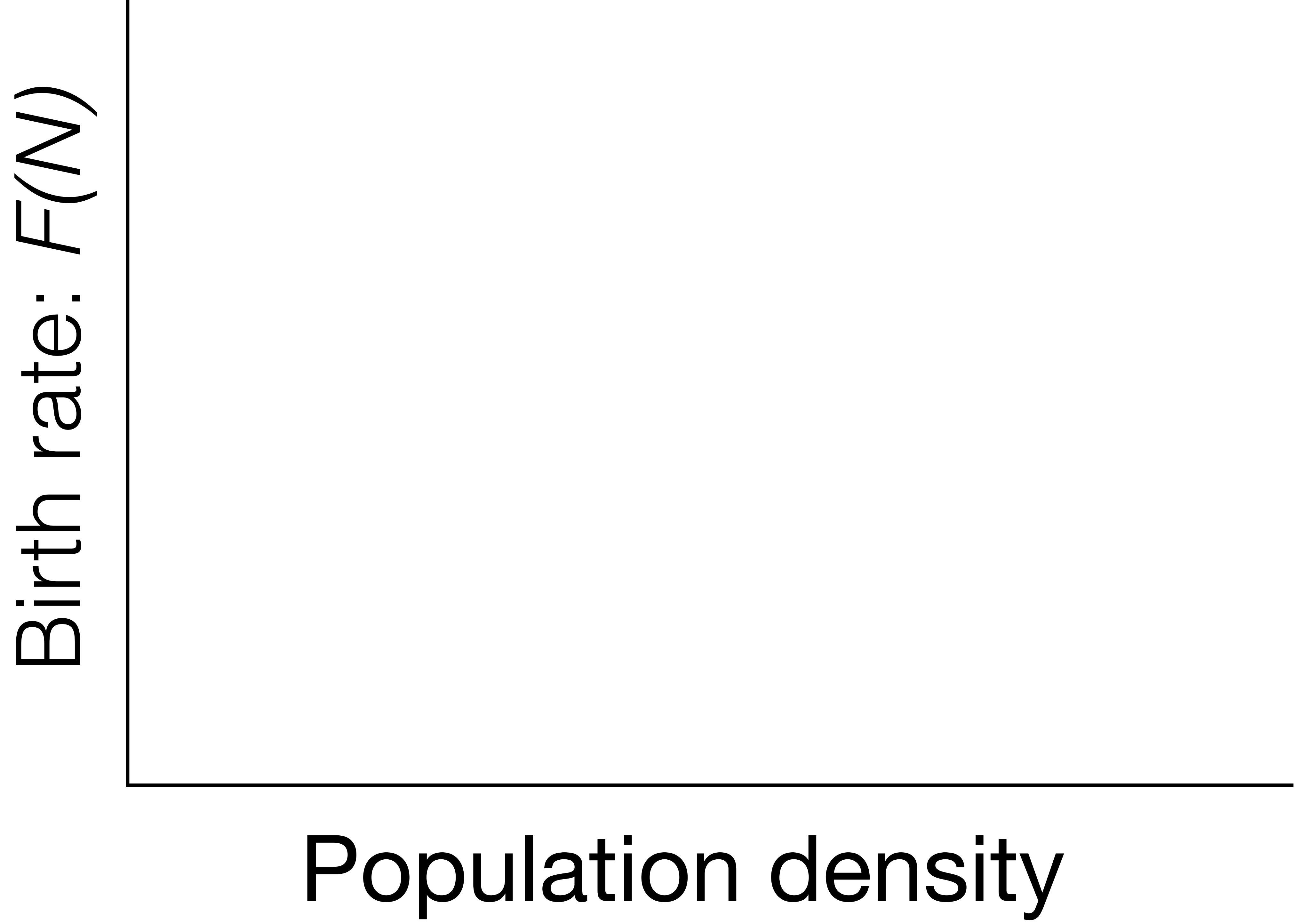
$$\frac{\mathrm{d}N}{\mathrm{d}t} = [b - df(N)]N$$

$$F(N) = d + cN = df(N) \quad \leftrightarrow \quad f(N) = 1 + N/k$$

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \left[b - d\left(1 + \frac{N}{k}\right)\right]N$$

$$\bar{N} = k\frac{b-d}{d} = k(R_0 - 1)$$





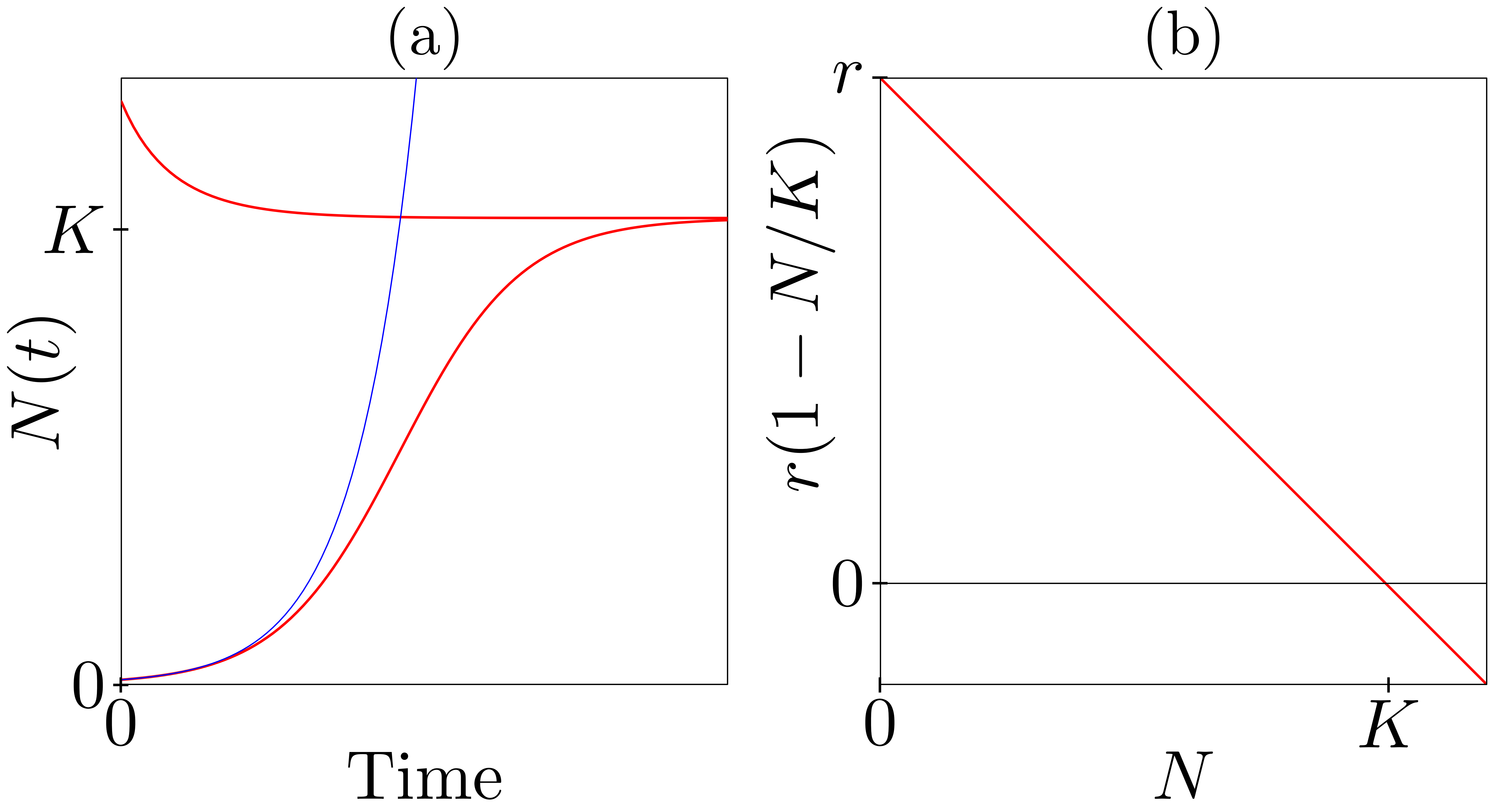
$$\frac{dN}{dt} = [bf(N) - d]N$$

$$F(N) = b - cN = bf(N) \quad \leftrightarrow \quad f(N) = 1 - N/k$$

$$\frac{dN}{dt} = \left[b\left(1 - \frac{N}{k}\right) - d\right]N$$

$$\bar{N} = k\left(1 - \frac{d}{b}\right) = k\left(1 - \frac{1}{R_0}\right)$$

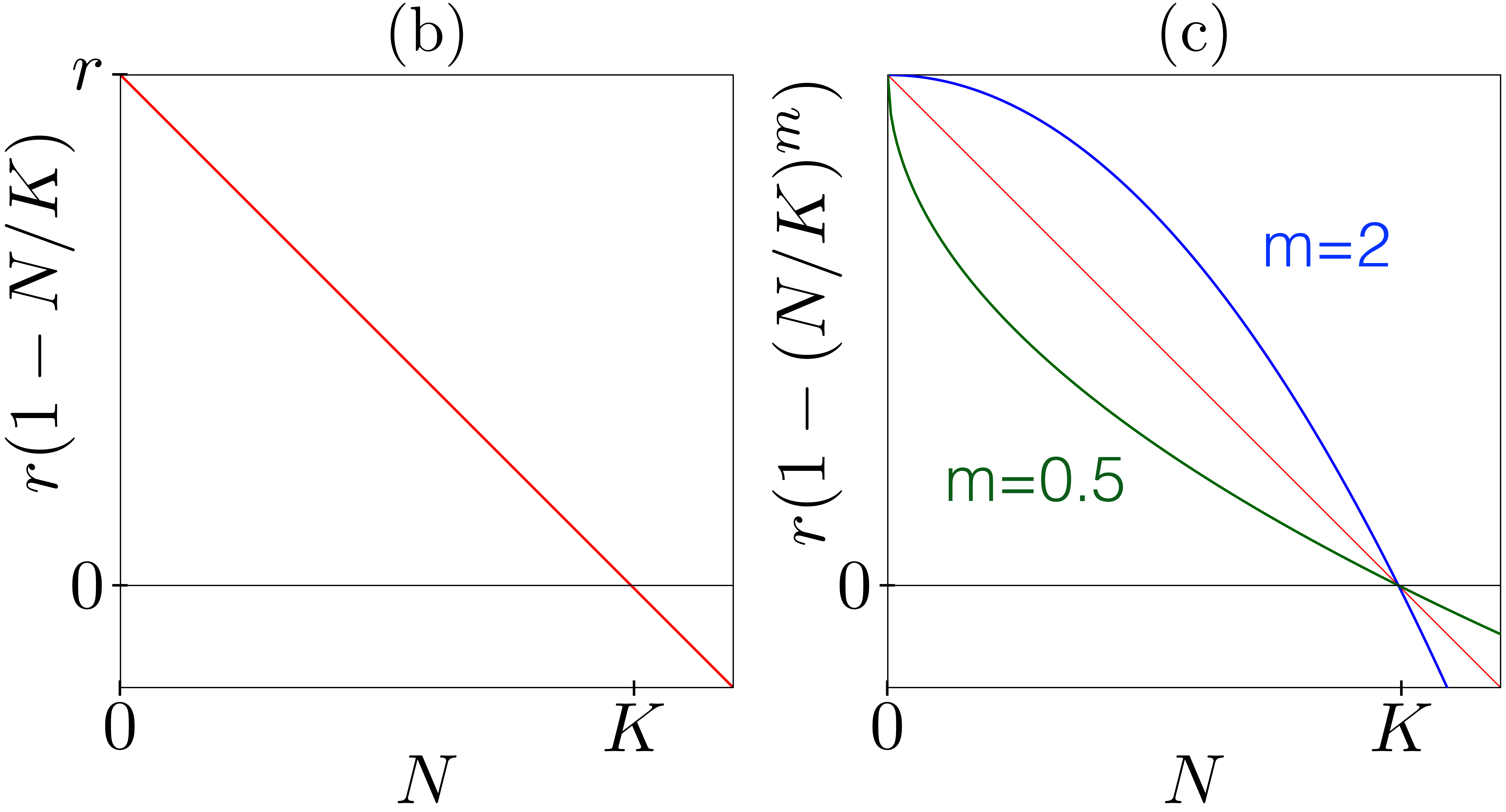
Logistic growth: $\frac{\mathrm{d}N}{\mathrm{d}t} = rN(1 - N/K)$, with solution $N(t) = \frac{KN(0)}{N(0) + \mathrm{e}^{-rt}(K - N(0))}$



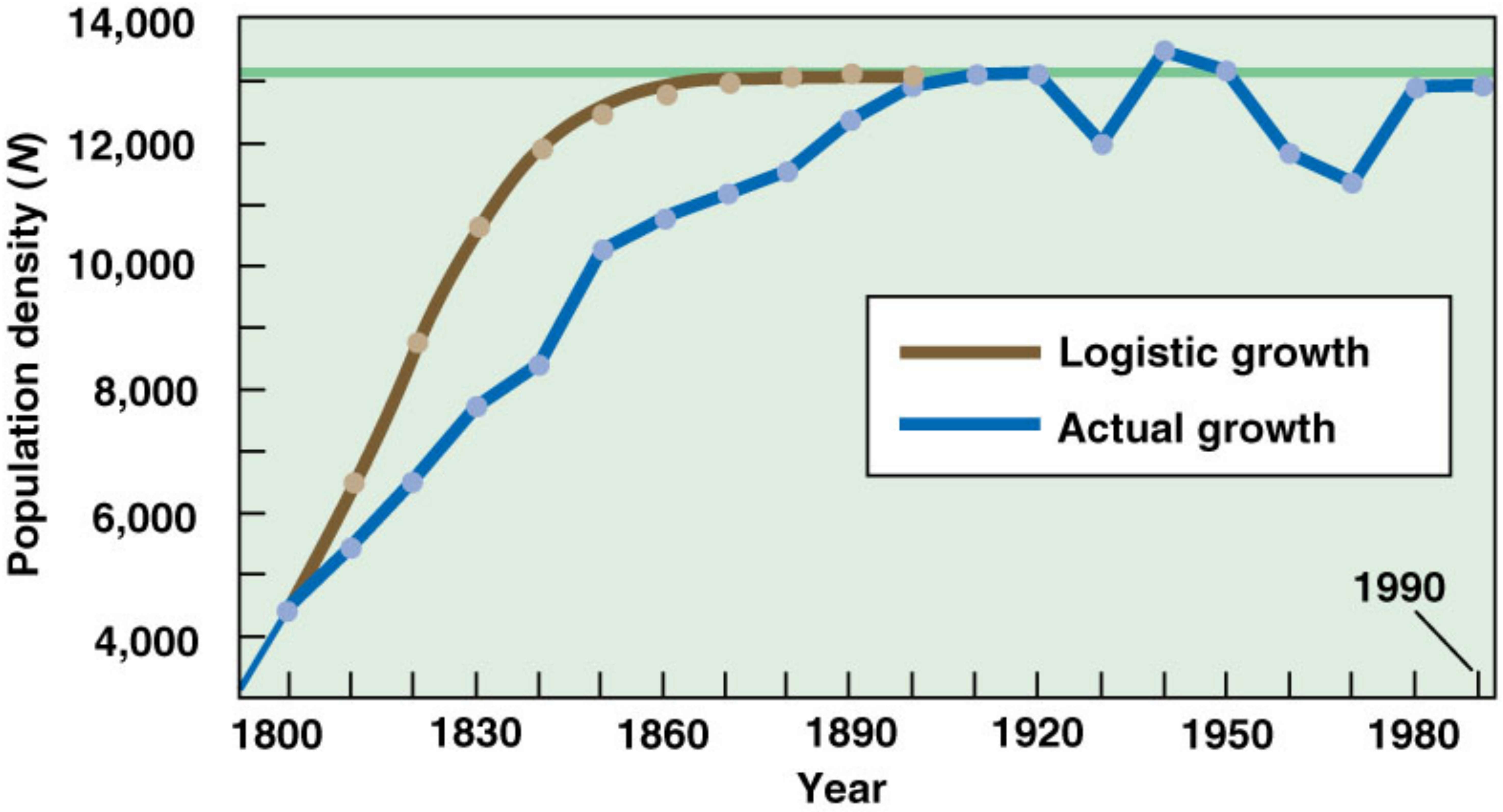
$$\frac{\mathrm{d}N}{\mathrm{d}t} = \left[b - d\left(1 + \frac{N}{k}\right)\right]N$$

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \left[b\left(1 - \frac{N}{k}\right) - d\right]N$$

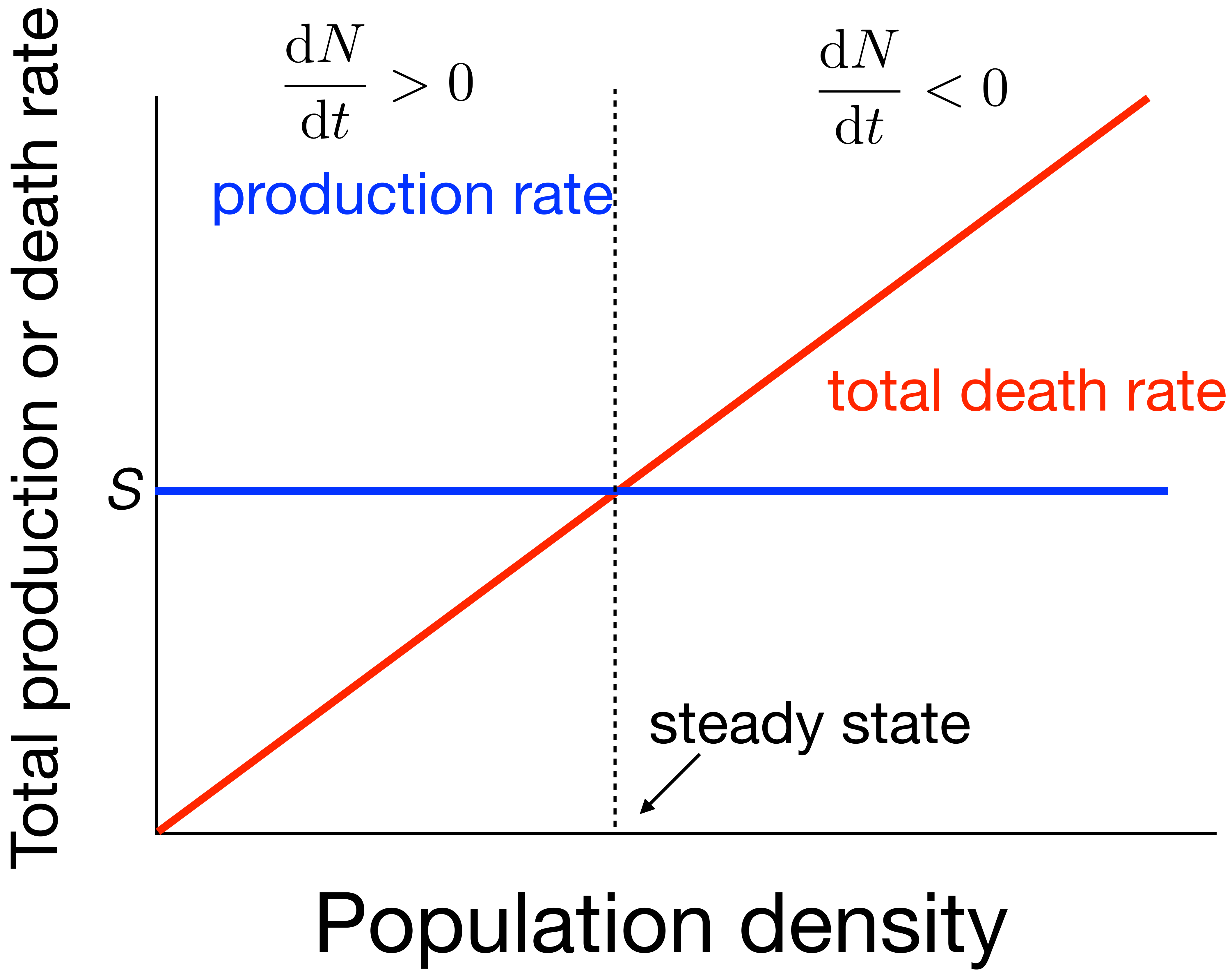
Generalized logistic growth: $\frac{\mathrm{d}N}{\mathrm{d}t} = rN(1-(N/K)^m)$, with $N(t) = \frac{K}{[1-(1-[K/N(0)]^m)\mathrm{e}^{-rmt}]^{1/m}}$

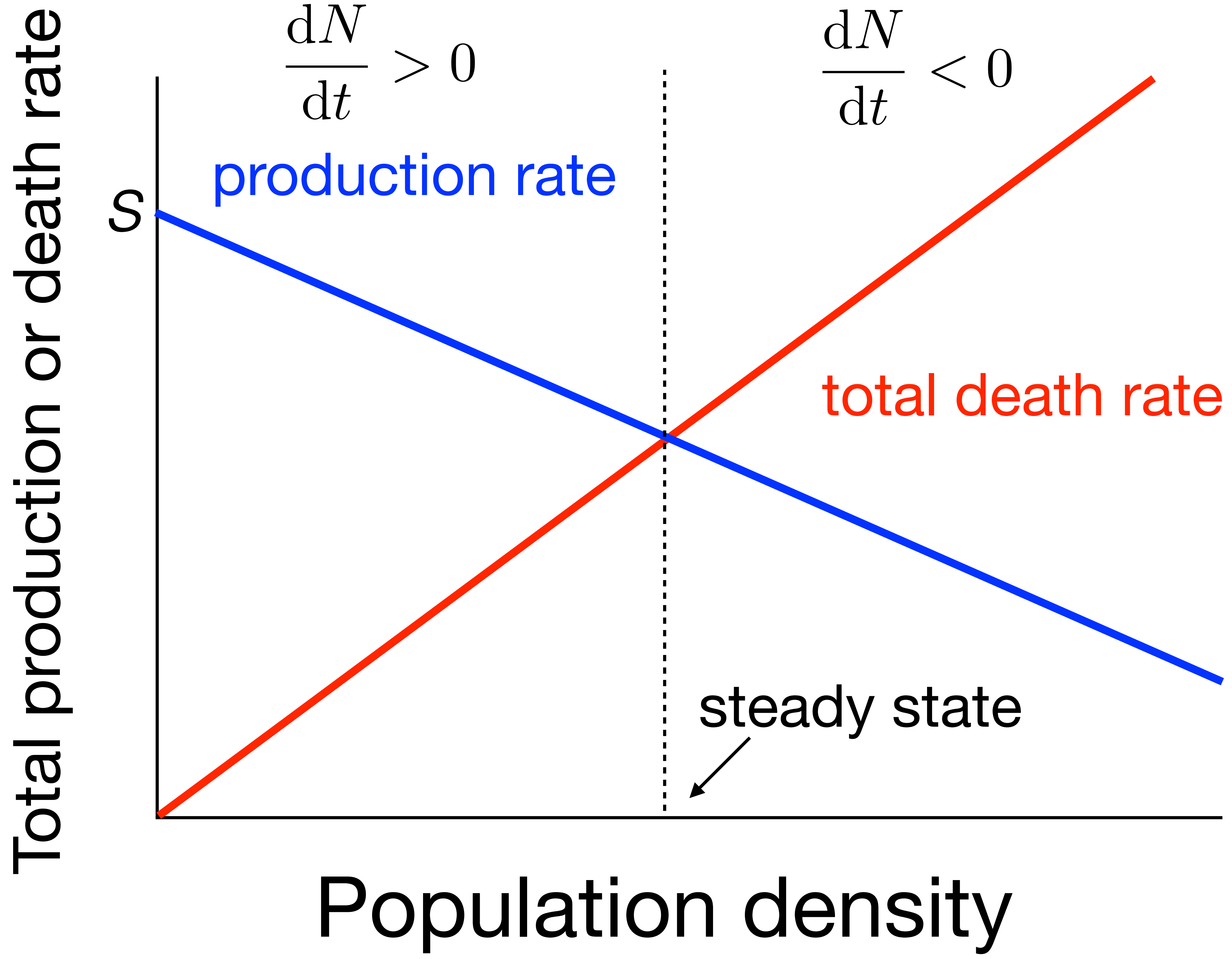


Human logistic growth



Human population in Monroe Country, West Virginia



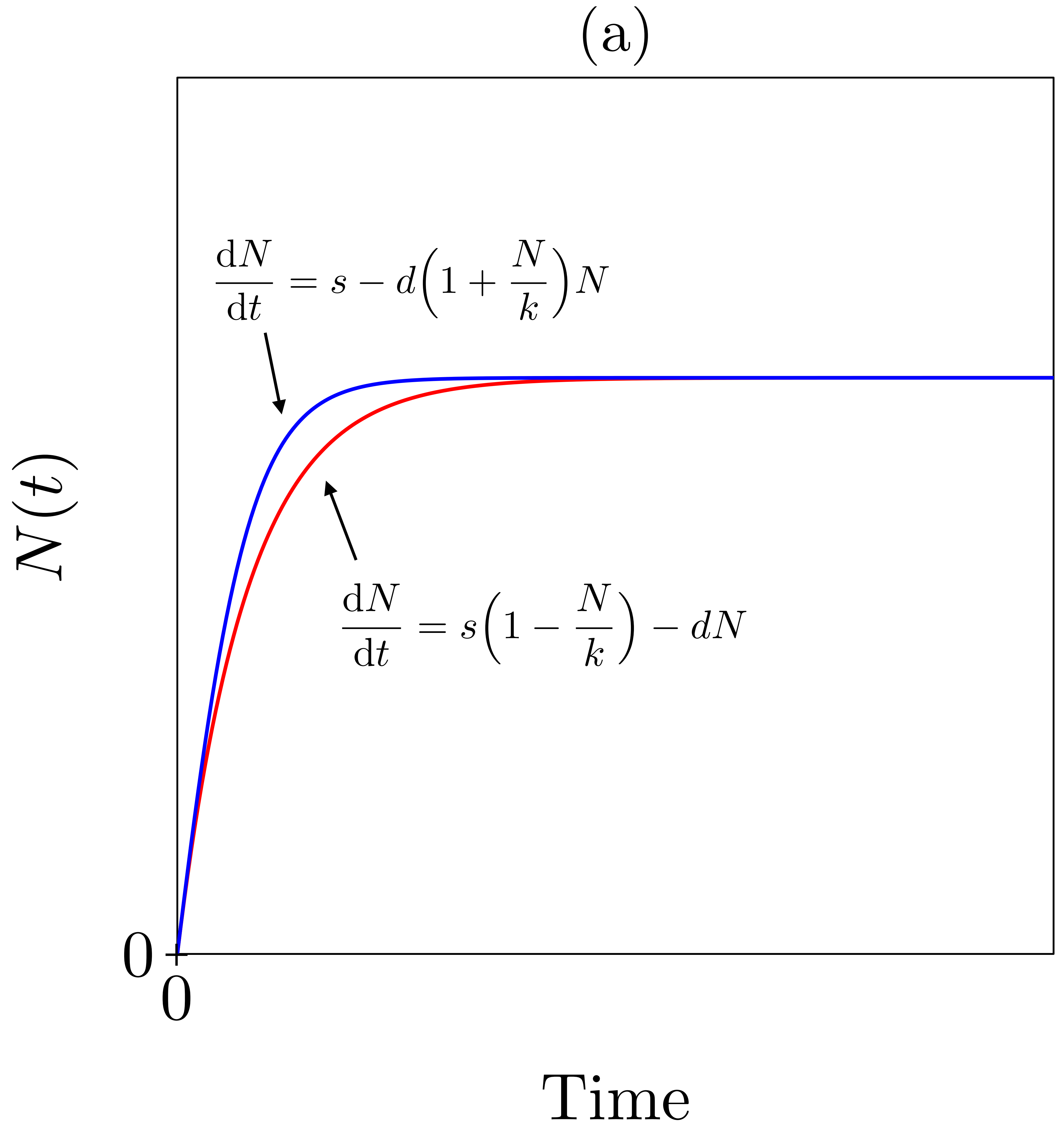


$$\frac{\mathrm{d}N}{\mathrm{d}t} = s - d\left(1 + \frac{N}{k}\right)N$$

$$\bar{N} = \frac{-dk \pm \sqrt{dk(dk + 4s)}}{2d}$$

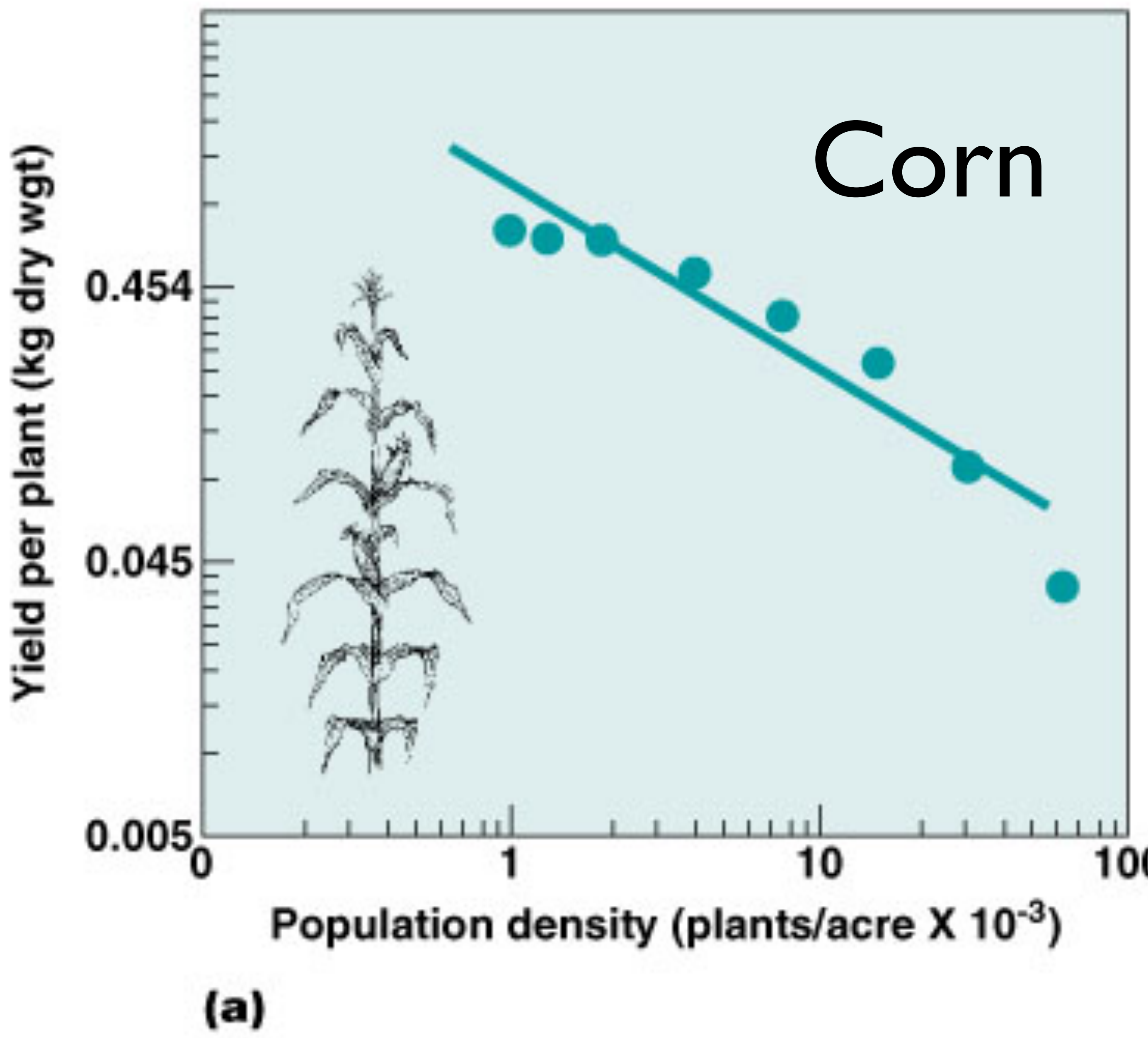
$$\frac{\mathrm{d}N}{\mathrm{d}t} = s\left(1 - \frac{N}{k}\right) - dN$$

$$\bar{N} = \frac{sk}{dk + s}$$

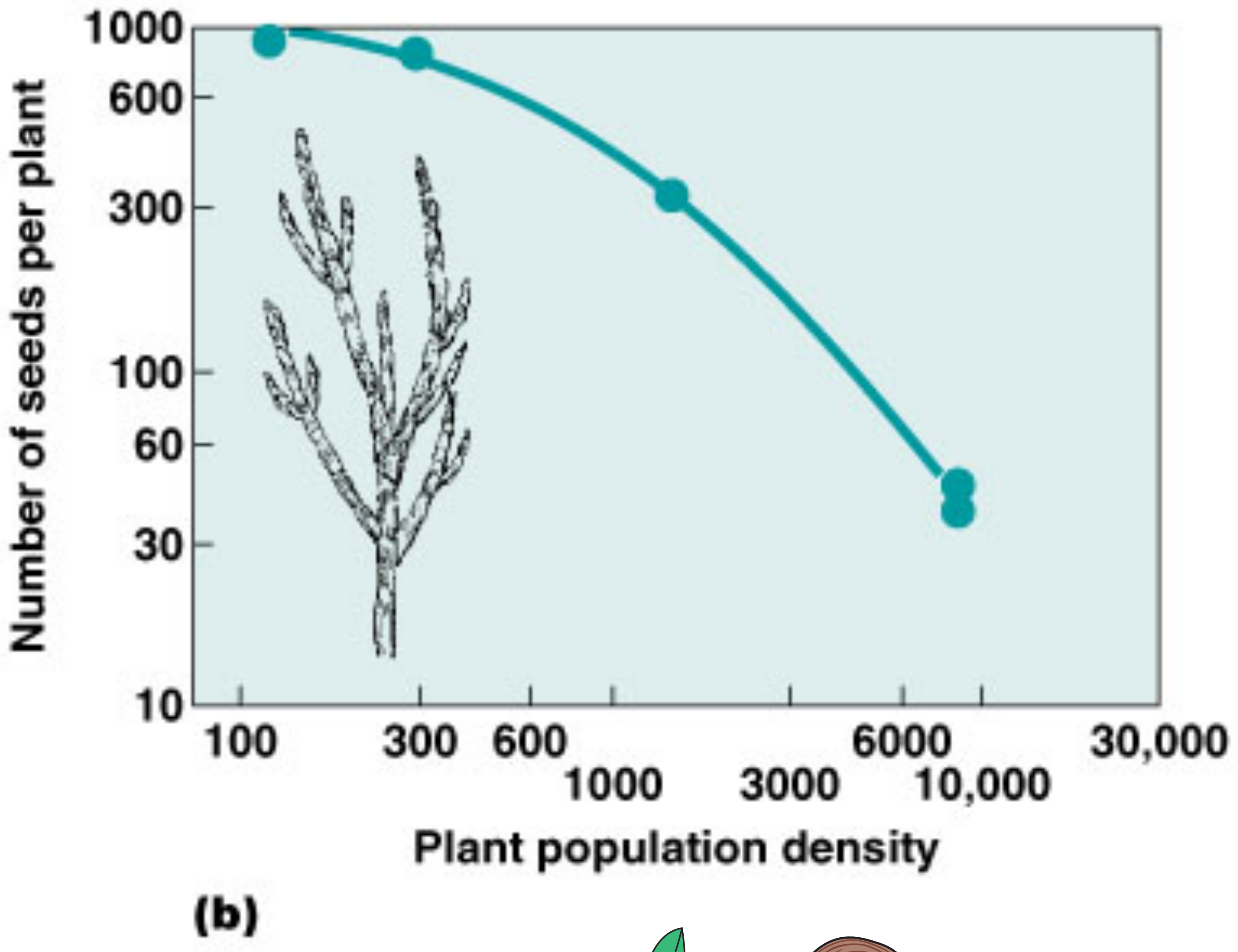


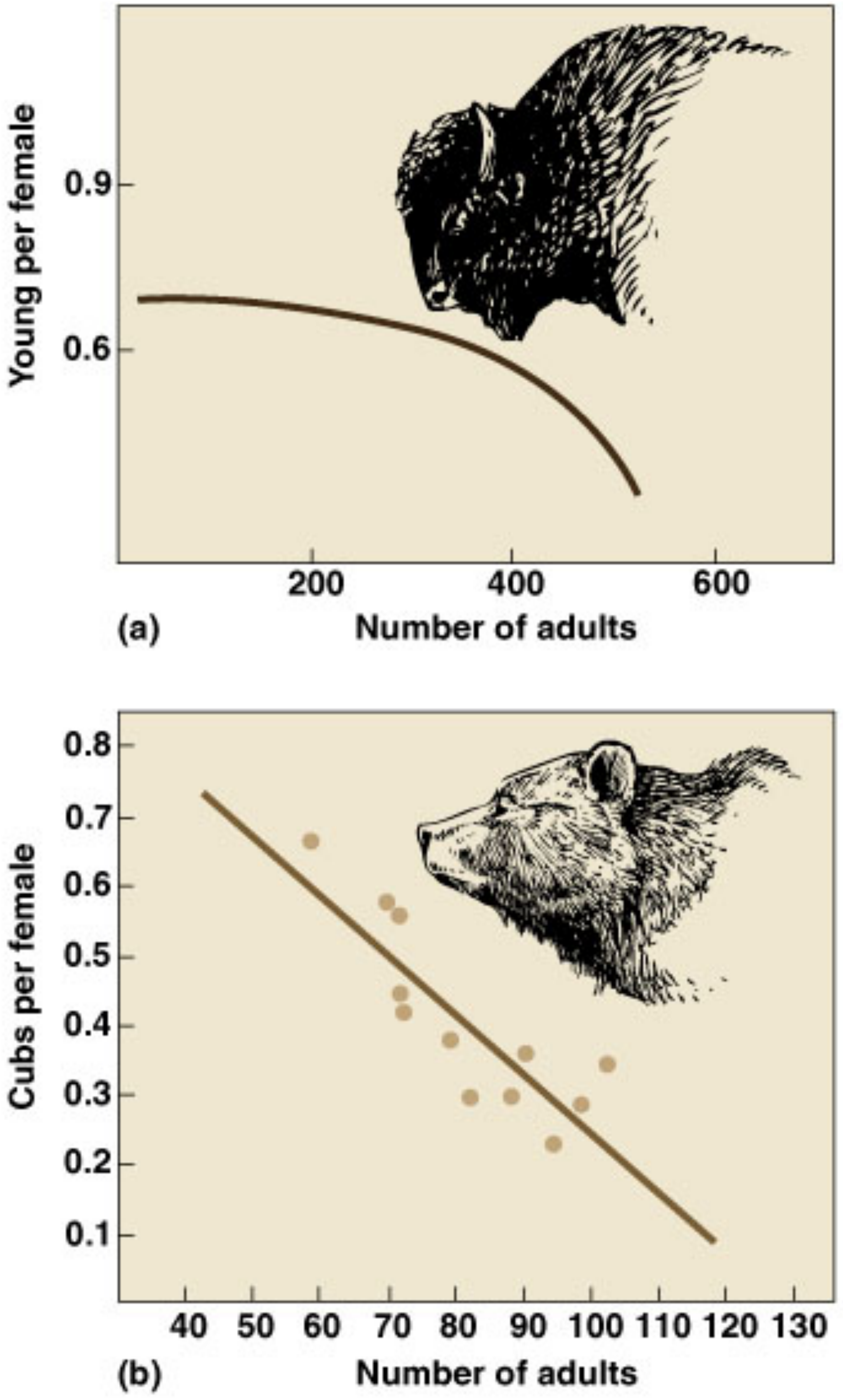
(a)
$\frac{\mathrm{d}N}{\mathrm{d}t} = s - d\left(1 + \frac{N}{k}\right)N$
$\frac{\mathrm{d}N}{\mathrm{d}t} = s\left(1 - \frac{N}{k}\right) - dN$
$N(t)$
0
0
Time

Density dependent birth is not always a linear function of *N*

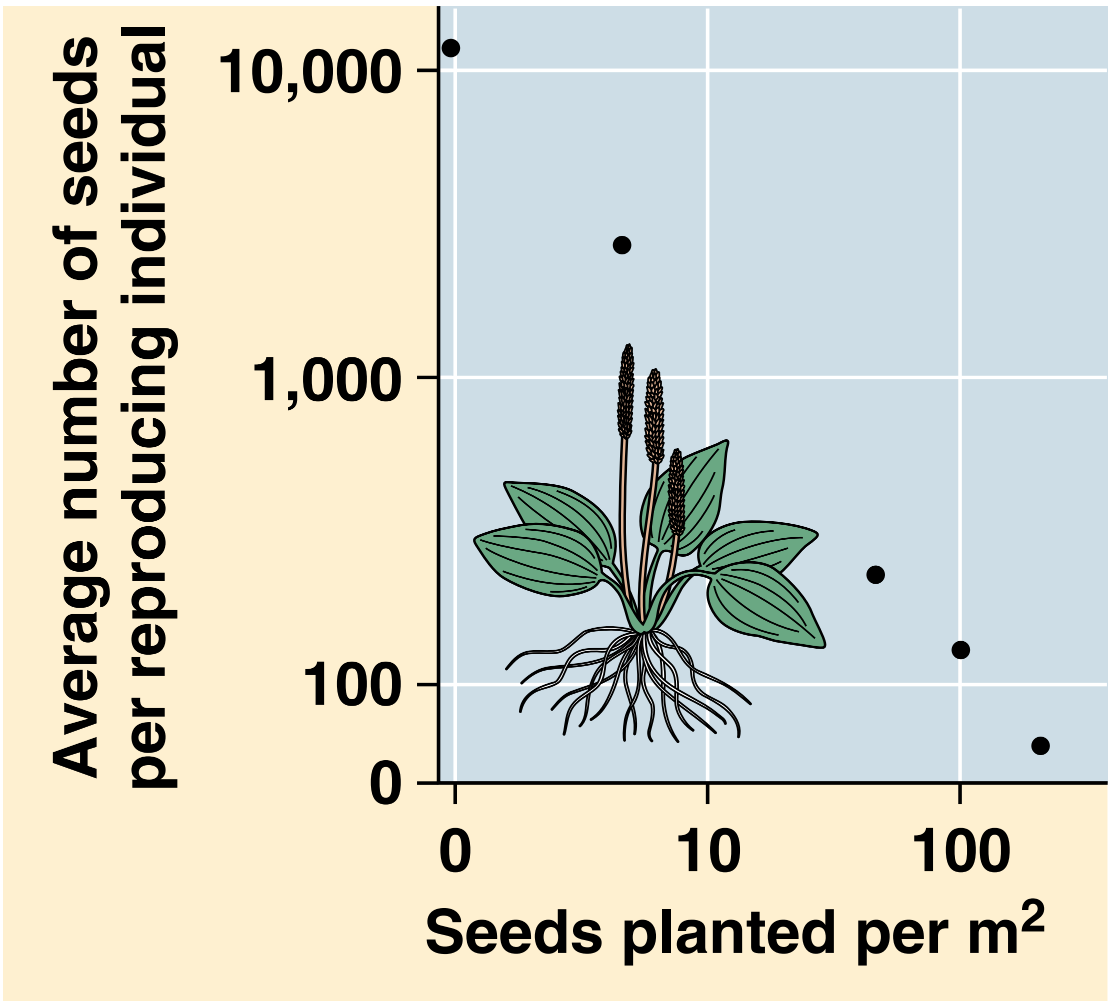


Salicornia

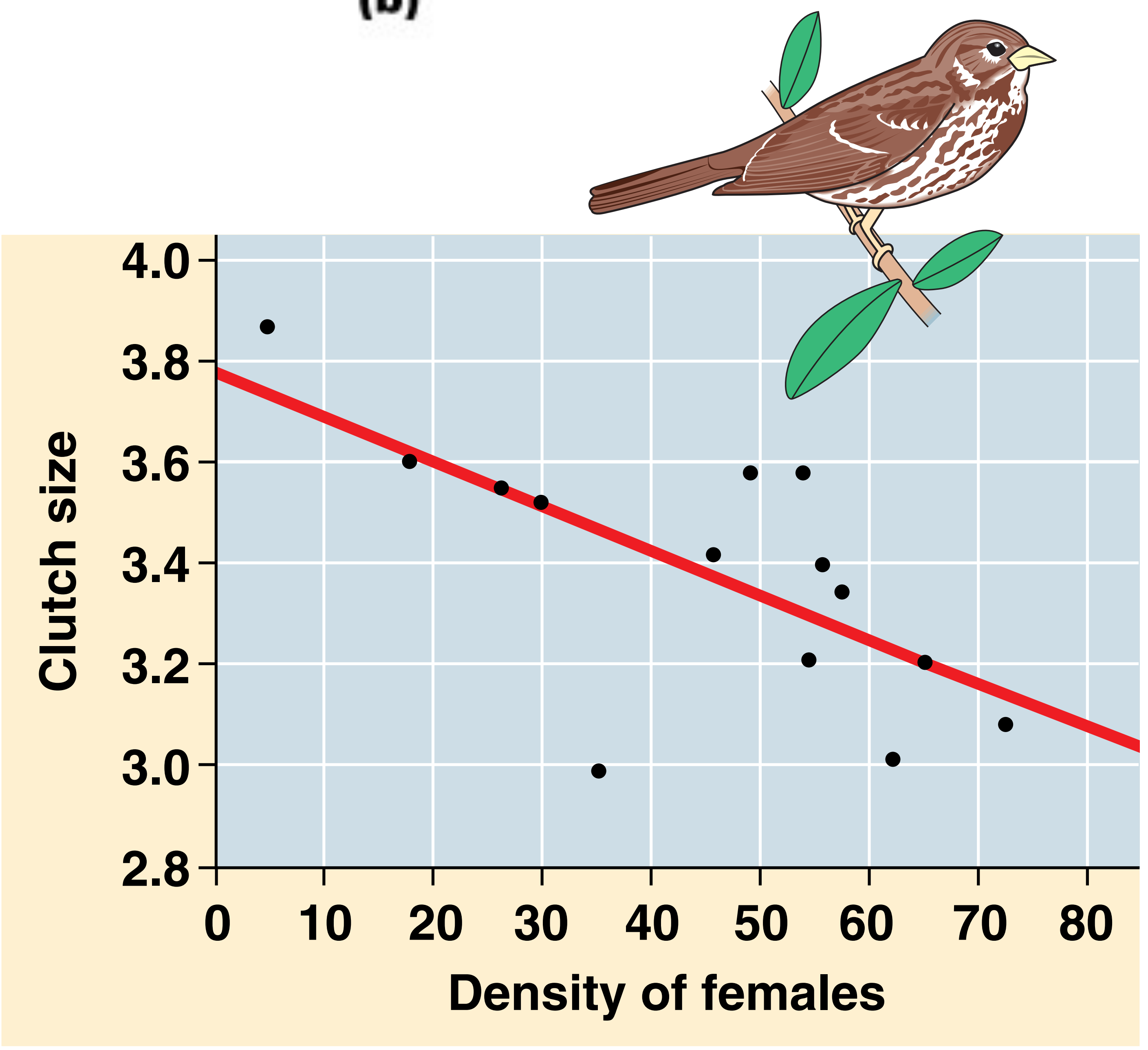




Grizzly bear



(a) Plantain



(b) Song sparrow

Non-linear density dependence

$$f(x) = \max(0, 1 - [x/k]^n)$$

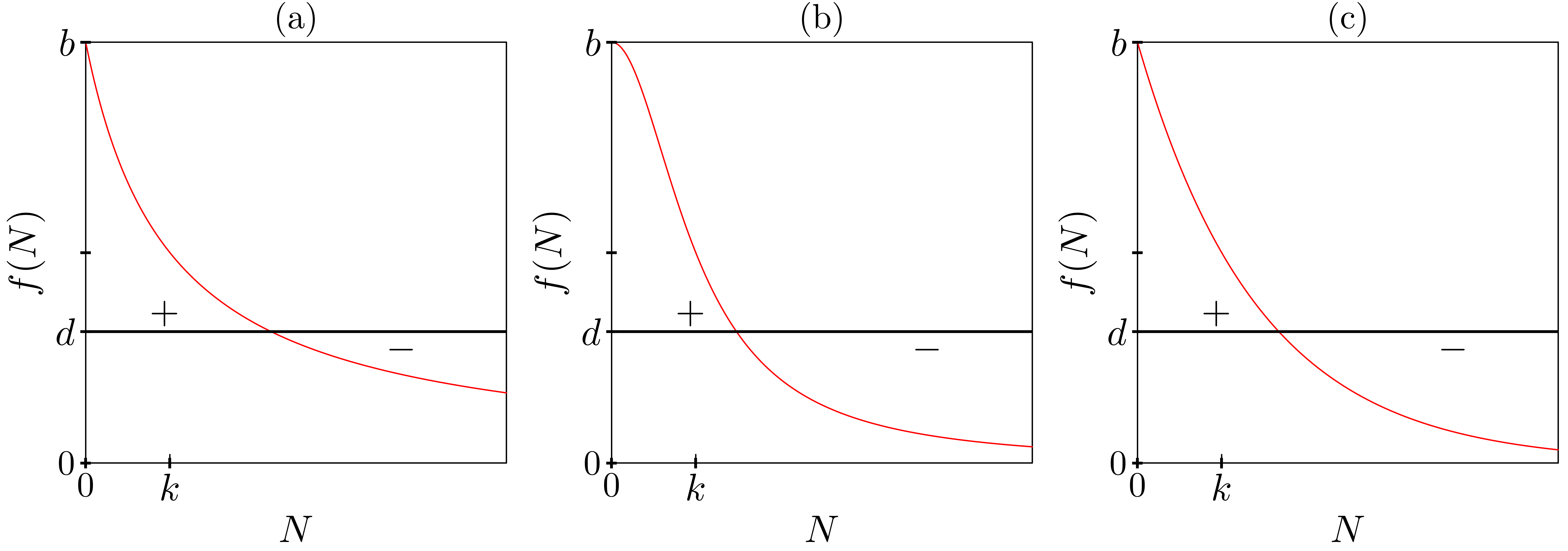
$$f(x) = \min(1, [x/k]^n)$$

$$f(x) = \frac{x^n}{h^n + x^n}$$

$$g(x) = \frac{1}{1 + (x/h)^n}$$

$$f(x) = 1 - \mathrm{e}^{-\ln[2]x/h} \qquad g(x) = \mathrm{e}^{-\ln[2]x/h}$$

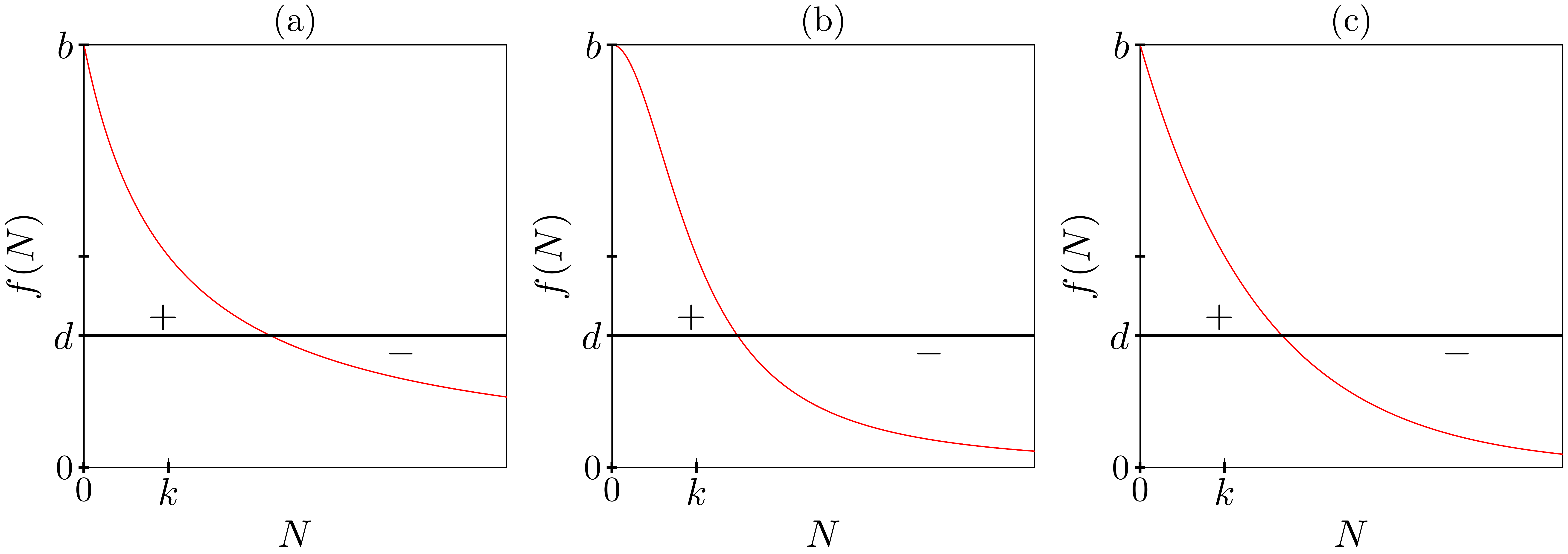
Non-linear density dependent birth rate



$$f(N) = \frac{1}{1 + N/k} \,, \quad f(N) = \frac{1}{1 + [N/k]^2} \quad \text{and} \quad f(N) = \mathrm{e}^{-\ln[2]N/k}$$

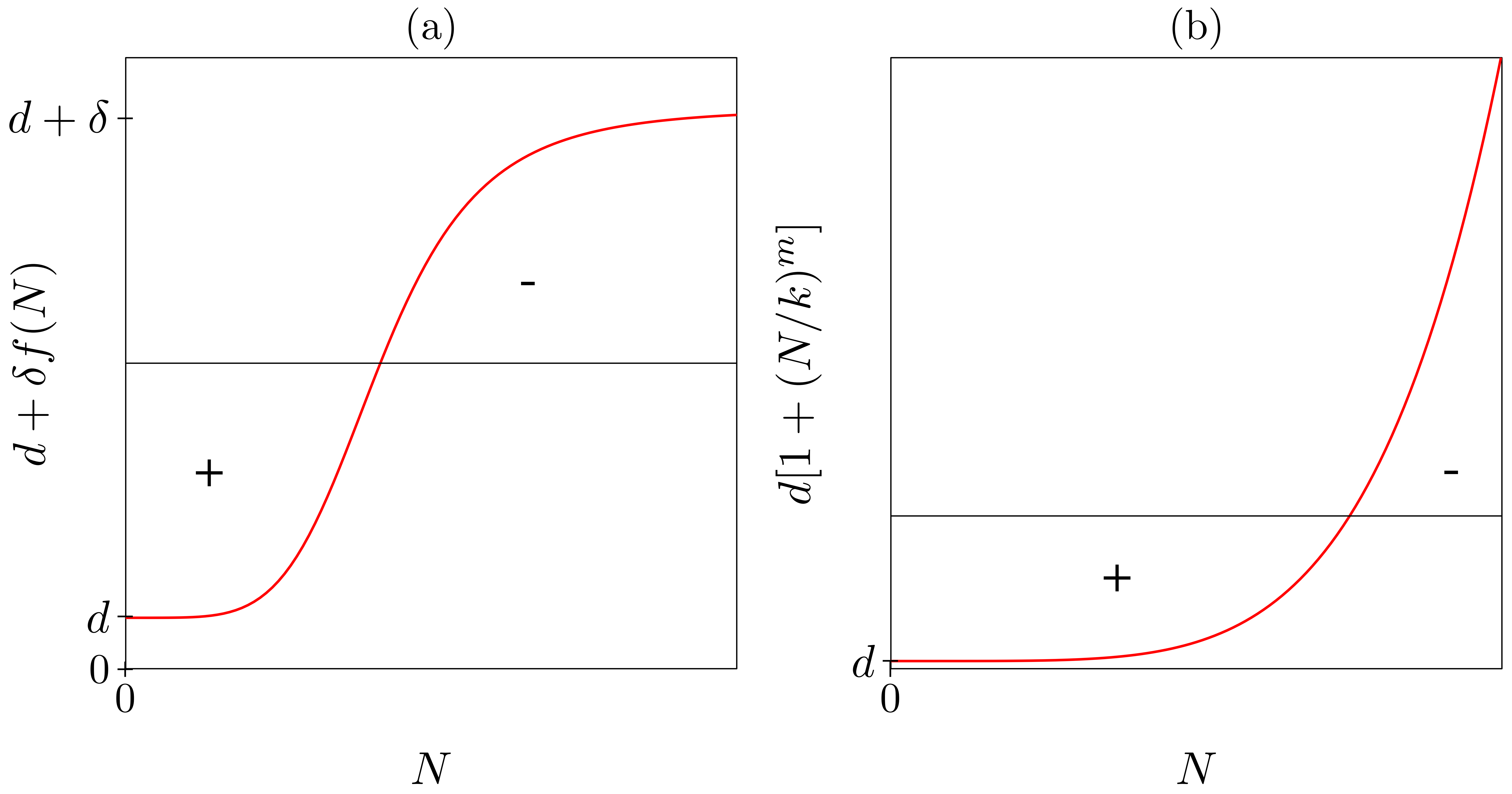
$$\frac{\mathrm{d}N}{\mathrm{d}t} = (bf(N) - d)N$$

$$\frac{\mathrm{d}N}{\mathrm{d}t} = (bf(N) - d)N \qquad f(N) = \frac{1}{1 + N/k}\,, \quad f(N) = \frac{1}{1 + [N/k]^2} \quad \text{and} \quad f(N) = \mathrm{e}^{-\ln[2]N/k}$$



Function	$f(0)$	$f(k)$	$f(\infty)$	R_0	Carrying capacity	Eq.
$f(N) = \max(0, 1 - [N/k]^m)$	1	0	0	b/d	$\bar{N} = k\sqrt[m]{1 - 1/R_0}$	(3.12)
$f(N) = 1/(1 + N/k)$	1	0.5	0	b/d	$\bar{N} = k(R_0 - 1)$	(3.14)
$f(N) = 1/(1 + [N/k]^2)$	1	0.5	0	b/d	$\bar{N} = k\sqrt{R_0 - 1}$	(3.14)
$f(N) = \mathrm{e}^{-\ln[2]N/k}$	1	0.5	0	b/d	$\bar{N} = (k/\ln[2])\ln[R_0]$	(3.14)

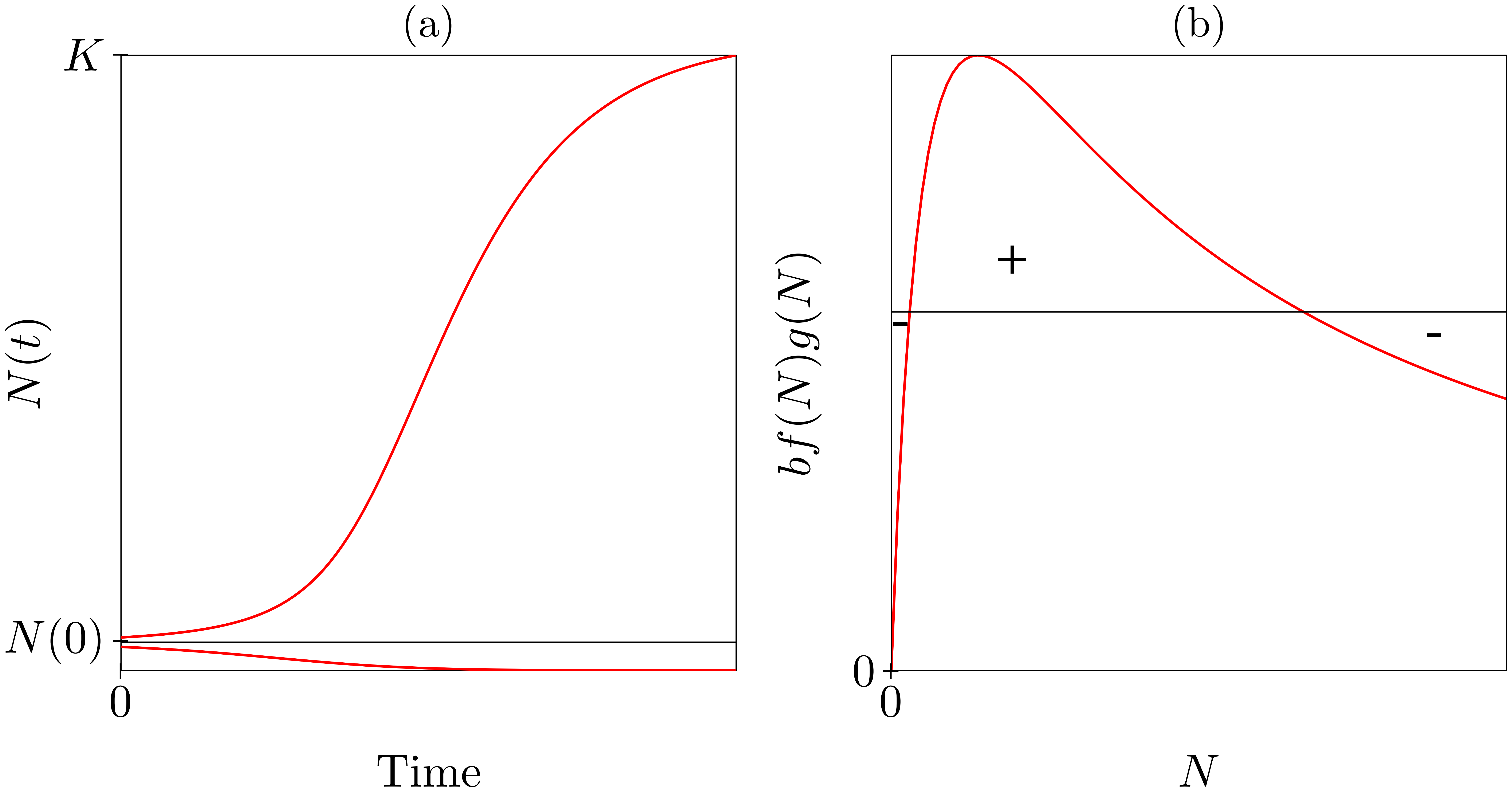
Non-linear density dependent death rate



$$\frac{\mathrm{d}N}{\mathrm{d}t} = [b - d - \delta f(N)]N$$

$$\frac{\mathrm{d}N}{\mathrm{d}t} = (b - d[1 + (N/k)^m])N$$

Positive density dependence



$$\frac{\mathrm{d}N}{\mathrm{d}t} = (bf(N)g(N) - d)N \quad \text{or} \quad \frac{\mathrm{d}N}{\mathrm{d}t} = (bg(N) - d[1 + (N/k)^m])N \ ,$$

Regression to the mean

```
n <- 100; data <- rnorm(n,1,0.1);hist(data)
N <- data[1:(n-1)]; r <- (data[2:n]-N)/N
plot(N,r,type="p")
lm(r~N,as.data.frame(cbind(N,r)))
```

Cell division takes time

Conventional ODE: $$\mathrm{d}N/\mathrm{d}t = (p-d)N$$

Smith-Martin model (first ignoring death):

$$\frac{\mathrm{d}A(t)}{\mathrm{d}t} = 2pA_{t-\Delta} - pA(t) \quad \text{and} \quad \frac{\mathrm{d}B(t)}{\mathrm{d}t} = pA(t) - pA_{t-\Delta}$$

Smith-Martin model with death:

$$\frac{\mathrm{d}A(t)}{\mathrm{d}t} = 2pA_{t-\Delta}\mathrm{e}^{-d\Delta} - (p+d)A(t) \quad \text{and} \quad \frac{\mathrm{d}B(t)}{\mathrm{d}t} = pA(t) - dB(t) - pA_{t-\Delta}\mathrm{e}^{-d\Delta}$$

Time delays implemented as many small steps

Smith-Martin model with death:

$$\frac{\mathrm{d}A(t)}{\mathrm{d}t} = 2pA_{t-\Delta}\mathrm{e}^{-d\Delta} - (p+d)A(t) \quad \text{and} \quad \frac{\mathrm{d}B(t)}{\mathrm{d}t} = pA(t) - dB(t) - pA_{t-\Delta}\mathrm{e}^{-d\Delta}$$

Smooth the time delay by many (*n*) small steps:

$$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{2n}{\Delta}B_n - (p+d)A\,, \quad \frac{\mathrm{d}B_1}{\mathrm{d}t} = pA - \left(d + \frac{n}{\Delta}\right)B_1 \quad \text{and} \quad \frac{\mathrm{d}B_i}{\mathrm{d}t} = \frac{n}{\Delta}\left(B_{i-1} - B_i\right) - dB_i$$